

2016.

Set No. : 1

Question Booklet No.

RET/16/TEST-B**895****Mathematics (Science)**

(To be filled up by the candidate by blue/black ball point pen)

Roll No.

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Roll No. (Write the digits in words)

Serial No. of OMR Answer Sheet

Day and Date

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 30 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall *except the Admit Card without its envelope.*
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.*
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. *On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.*
6. *No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.*
7. *Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.*
8. *This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.*
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. *Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks)*
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. *Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.*
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 24

39.

RET/16/TEST-B

895/Mathematics (Science)

ROUGH WORK

रफ़ कार्य

Research Entrance Test-2016

No. of Questions : 50

प्रश्नों की संख्या : 50

Time : 2 Hours

Full Marks : 200

समय : 2 घण्टे

पूर्णाङ्क : 200

Note: (1) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

इस प्रश्न पुस्तिका में **40** वस्तुनिष्ठ व **10** लघु उत्तरीय प्रश्न हैं।

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

अधिकाधिक वस्तुनिष्ठ प्रश्नों को हल करने का प्रयत्न करें। प्रत्येक वस्तुनिष्ठ प्रश्न **3 (तीन)** अंकों का है। प्रत्येक गलत उत्तर के लिए **1 (एक)** अंक काटा जायेगा। प्रत्येक अनुत्तरित प्रश्न का प्राप्तांक शून्य होगा। यदि वस्तुनिष्ठ प्रश्नों के एकाधिक वैकल्पिक उत्तर सही उत्तर के निकट प्रतीत हों, तो निकटतम सही उत्तर दें।

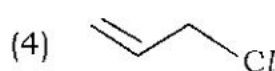
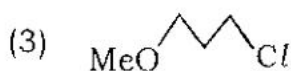
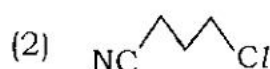
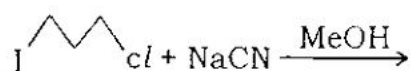
(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

केवल **5 (पाँच)** लघुउत्तरीय प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **16 (सोलह)** अंकों का है तथा उनका उत्तर **150-200** शब्दों के बीच होना चाहिए। इसके लिए इस पुस्तिका में लगे हुए सादे **5 (पाँच)** पृष्ठों का ही उपयोग आवश्यक है। प्रत्येक प्रश्न का उत्तर एक नए पृष्ठ से, प्रश्न संख्या लिखकर शुरू करें।

01. Which is not true for reactions by the S_N2 mechanism ?

- (1) proceeds through a backside attack and results in inversion
- (2) tends to proceed with weak nucleophiles solvents like CH_3OH , H_2O , CH_3CH_2OH .
- (3) rate of reaction proceeds from primary (fastest) > secondary >> tertiary (slowest)
- (4) occurs in one step

02. Which is the main product of the following reaction ?



03. Which of the following conditions is necessary for a reaction to be spontaneous ?

(1) $\Delta S_{sur} > 0$

(2) $\Delta S_{sys} > 0$

(3) $\Delta S_{sur} + \Delta S_{sys} > 0$

(4) $\Delta S_{sur} + \Delta S_{sys} < 0$

04. Dead organs are generally stored in formalin. Formalin is :

(1) aqueous formaldehyde

(2) aqueous ferrous sulphate

(3) aqueous formic acid

(4) aqueous ferric alum

05. Regarding "carbon credits", which one of the following statement is **not** correct :

- (1) The carbon credit system was ratified in conjunction with the Kyoto Protocol.
- (2) Carbon credits are awarded to countries or groups that have reduced greenhouse gases below their emission quota.
- (3) The goal of the carbon credit system is to limit the increase of carbon dioxide emission.
- (4) Carbon credits are traded at a price fixed from time to time by the United Nations Environment Programme.

06. Ball bearings are used in bicycles, cars, etc., because :

- (1) the actual area of contact between the wheel and axle is increased.
- (2) the effective area of contact between the wheel and axle is increased
- (3) the effective area of contact between the wheel and axle is reduced
- (4) the actual area of contact between the wheel and axle is reduced.

07. During respiration, energy is released. It is stored in the form of :

- (1) ADP (2) ATP (3) NADP (4) APP

08. Which of the following is known as Royal disease :

- | | |
|------------------------|----------------------|
| (1) Sickle cell anemia | (2) Haemophilia |
| (3) Alzheimers disease | (4) Colour blindness |

09. The xylem in plants is responsible for :

- | | |
|-------------------------|------------------------------|
| (1) transport of water | (2) transport of food |
| (3) transport of oxygen | (4) transport of amino acids |

10. Two wires, of the same material, have their lengths in the ratio 1:2 and their diameters in the ratio 2:1. If both are stretched separately by equal weights, the ratio of increase in their lengths, $L_1 : L_2$ would be :

- | | | | |
|---------|---------|---------|---------|
| (1) 1:2 | (2) 2:1 | (3) 1:8 | (4) 8:1 |
|---------|---------|---------|---------|

11. A sphere starts rotating under no forces about its centre with angular velocity components $2n$, n and $3n$ about its principal axes respectively. During rotation :

- (1) each angular velocity component remains same
- (2) only second angular velocity component remains same and other two change.
- (3) first and third angular velocity components become periodic
- (4) all the three components change but resultant of angular velocity remains same.

12. If q and p are generalized co-ordinate and momentum variable respectively, then.

$$(1) \quad p = \frac{\partial T}{\partial q}$$

$$(2) \quad p = \frac{\partial V}{\partial q}$$

$$(3) \quad p = \frac{\partial L}{\partial q}$$

$$(4) \quad p = \frac{\partial S}{\partial q}$$

Where T, V, L and S are the Kinetic energy, Potential energy, Lagrange function and Hamilton's principal function respectively.

13. If transformation equations from one phase space (q_r, p_r) to another phase space (Q_r, P_r) are canonical, then :

$$(1) \quad \frac{\partial P_r}{\partial q_r} = \frac{\partial p_r}{\partial Q_r}$$

$$(2) \quad \frac{\partial P_r}{\partial Q_r} = \frac{\partial p_r}{\partial q_r}$$

$$(3) \quad \frac{\partial P_r}{\partial q_r} = -\frac{\partial p_r}{\partial Q_r}$$

$$(4) \quad \frac{\partial Q_r}{\partial P_r} = \frac{\partial p_r}{\partial q_r}$$

14. The family of curves given by $\phi = \text{constant}$ and $\psi = \text{constant}$ in fluid dynamics intersects at :

$$(1) \quad 45^\circ$$

$$(2) \quad 60^\circ$$

$$(3) \quad 90^\circ$$

$$(4) \quad 75^\circ$$

15. Streamlines and pathlines become same when the motion is :

$$(1) \quad \text{Steady}$$

$$(2) \quad \text{Unsteady}$$

$$(3) \quad \text{Turbulent}$$

$$(4) \quad \text{Larminar}$$

16. If U_{\max} is the maximum velocity in the plane Poiseuille flow, then the average velocity in the flow is given by

(1) $\frac{1}{3} U_{\max}$

(2) $\frac{2}{3} U_{\max}$

(3) $\frac{3}{2} U_{\max}$

(4) $\frac{1}{2} U_{\max}$

17. The difference equation at the boundary point $x = 0$ of the boundary value problem $y'' - y = x$, $y'(0) = 0$, $y(1) = 1$ by using the central difference schemes for the derivatives when width of each sub-interval is 0.25 is :

(1) $32 y_0 + 33 y_1 = 0$

(2) $33 y_0 - 32 y_1 = 0$

(3) $33 y_0 + 32 y_1 = 0$

(4) $32 y_0 - 33 y_1 = 0$

18. Initial value problem $y'' + y = 0$, $y(0) = y'(0) = 0$ is equivalent to the integral equation :

(1) $y(x) = - \int_0^x (x-t) y(t) dt$

(2) $y(x) = \int_0^x (x-t) y(t) dt$

(3) $y(x) = - \int_0^x (x+t) y(t) dt$

(4) $y(x) = \int_0^x (x+t) y(t) dt$

19. The solution of partial differential equation $\frac{\partial u}{\partial t} + 2 \frac{\partial u}{\partial x} + 3u = 0$, with $u(x, 0) = \sin x$ is :

(1) $\sin(x - 2t)$

(2) $e^{-t} \sin(x - 2t)$

(3) $e^{-3t} \sin(x - 2t)$

(4) $e^{2t} \sin x$

20. The nature of partial differential equation

$$\cos^2 x \frac{\partial^2 u}{\partial x^2} + \sin 2x \frac{\partial^2 u}{\partial x \partial y} + \cos^2 x \frac{\partial^2 u}{\partial y^2} = x^2 \text{ is :}$$

- (1) parabolic (2) hyperbolic
(3) elliptic (4) circular

21. The number of zeros of the polynomial $p(z) = z^4 - 4z^3 + z - 1 = 0$ lying inside the unit disc is :

- (1) 4 (2) 3 (3) 2 (4) 1

22. The value of integral $\int_{z=n} \frac{dz}{z^2 \sin z}$ is :

- (1) $\frac{\pi i}{3}$ (2) $\frac{2\pi i}{3}$ (3) πi (4) 0

23. A finite abelian group of prime power order is internal direct product of :

- (1) its subgroups (2) its normal subgroups
(3) cyclic groups (4) subgroups of prime order

24. Total number of elements of order 5 in the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$ is :

- (1) 25 (2) 24 (3) 5 (4) 6

25. $U(15)$, the set of all integers less than 15 and relative prime to 15 is a group under multiplication module 15. Number of cyclic subgroups of $U(15)$ is :

- (1) 2 (2) 3 (3) 4 (4) 6

26. The number of cyclic subgroups of order 15 in the group $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$ is :

- (1) 6 (2) 5 (3) 10 (4) 11

27. If A° and B° denote interior of A and B respectively, then :

- (1) $A^\circ \cup B^\circ = (A \cup B)^\circ$ (2) $A^\circ \cup B^\circ = (A \cap B)^\circ$
 (3) $A^\circ \cup B^\circ \supseteq (A \cup B)^\circ$ (4) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$

28. If Ω is a convex subset of \mathbb{R}^n , then which one of the following is false :

- (1) $\text{int } \Omega$ is a convex set
 (2) $\bar{\Omega}$ is a convex set
 (3) $\overline{\text{int } \Omega} = \bar{\Omega}$
 (4) for any $a \in \text{int } \Omega$ and $b \in \bar{\Omega}$, $[a, b] \subset \text{int } \Omega$.

29. Let $C_{[a,b]}$ be a vector space of all real valued continuous functions defined on closed interval $[a,b]$, $0 < a < b < \infty$, if

$$\text{for } f \in C_{[a,b]}, \|f\|_3 = \left(\int_a^b |f(x)|^3 dx \right)^{1/3}, \text{ then}$$

- (1) $(C_{[a,b]}, \|\cdot\|_3)$ is a normed linear space but not a Banach space over a set of all real numbers \mathbb{R}
 - (2) $(C_{[a,b]}, \|\cdot\|_3)$ is a Banach space over \mathbb{R}
 - (3) $(C_{[a,b]}, \|\cdot\|_3)$ is not a normed linear space over \mathbb{R}
 - (4) In $(C_{[a,b]}, \|\cdot\|_3)$ every absolutely convergent series is convergent in \mathbb{R} .
30. Which one of the following is false :

$$(1) \quad f(x) = \begin{cases} 1 & \text{for } -1 \leq x \leq 1, x \neq 0 \\ 2 & \text{for } x = 0 \end{cases}$$

is quasi convex

$$(2) \quad f(x) = \begin{cases} \frac{|x|}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

is quasi convex

$$(3) \quad f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } 1 < x \leq 2 \end{cases}$$

is quasi convex

$$(4) \quad f(x) = \begin{cases} x & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } 1 < x \leq 2 \end{cases}$$

is not semi-strictly quasi convex

31. If C is a closed convex cone in \mathbb{R}^n , then which one of the following is false :

- (1) C^* is a closed convex cone.
- (2) $x \in C \Leftrightarrow \alpha^T x \geq 0 \quad \forall \alpha \in C^*$
- (3) $x \in \text{int } C \Leftrightarrow \alpha^T x > 0 \quad \forall \alpha \in C^* \setminus \{0\}$
- (4) $\alpha \in \text{int } C^* \Leftrightarrow \alpha^T x \geq 0 \quad \forall x \in C \setminus \{0\}$

32. The radius of convergence of power series $\sum_{n=0}^{\infty} n! z^n$ is :

- (1) 1
- (2) e
- (3) $\frac{1}{e}$
- (4) 0

33. If $a > 0, b > 0, 1 < p < \infty, \frac{1}{p^2} + \frac{1}{q^2} = 1$ and $ab = \frac{a^{p^2}}{p^2} + \frac{b^{q^2}}{q^2}$ then :

- (1) $a^{p^2} = b^{q^2}$
- (2) $a^{p^2} = b^{q^2}$
- (3) $a^{\frac{1}{p}} = b^{\frac{1}{q}}$
- (4) $a^{\frac{1}{p^3}} = b^{\frac{1}{q^2}}$

34. The series :

$$1 - 3 + 6 - 10 + 15 - \dots$$

- (1) is convergent but not absolutely convergent.
- (2) is absolutely convergent
- (3) oscillates finitely
- (4) oscillates infinitely

35. The value of $\int_0^{\infty} \frac{dx}{(1+x)x^{1/2}}$ is :

- (1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

36. The general linear group $GL(n, \mathbb{R})$ is differentiable manifold of dimension :

- (1) $\frac{n(n+1)}{2}$ (2) $\frac{n(n-1)}{2}$
 (3) n^2 (4) $n^2/2$

37. For any $x, y, z \in \chi(m)$ and $f, g \in C^1(m)$, $[f x, g y] =$

- (1) $fg[x, y] + f(xg)y - g(yf)x$ (2) $fg[x, y] + f(xg)y + g(yf)x$
 (3) $fg[x, y] + (xg)(yf) - (yf)(xg)$ (4) $fg[x, y] + (xg)(yf) + (yf)(xg)$

38. The integral curves for the vector field in \mathbb{R}^2 given

by $X = x^2 \frac{\partial}{\partial x^1} - (x^2)^3 \frac{\partial}{\partial x^2}$ is :

- (1) parabola (2) hyperbola
 (3) ellipse (4) circle

39. If $w = xdx + ydy + (x + z) dz$ be 1-form in \mathbb{R}^3 , then the exterior differential dw is:

- (1) $dx \wedge dy$ (2) $dx \wedge dz$
 (3) 0 (4) $dy \wedge dx$

40. If an open set $U \subset \mathbb{R}^n$ is diffeomorphic to an open set $V \subset \mathbb{R}^m$, then :

(1) $n = m$

(2) $n > m$

(3) $n < m$

(4) $n = m + 1$

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. Derive Hamilton's equations of motion from Hamilton-Jacobi theory in generalised system of rigid body motion.
2. Derive the equation of continuity in cylindrical coordinate system.
3. Use the Galerkin method to solve the boundary value problem $y'' + y = \frac{1}{4} (7x - 12x^2)$, $y(0) = 0$, $y(2) = 0$ by taking the approximate function as $c_1 x(x-2) + c_2 x^2(x-2)$.
4. Using the contour integration, evaluate the integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$.
5. (a) Prove that continuous image of a compact space is compact.
(b) Prove that continuous image of a connected space is connected.
6. Let G be a finite group and let $p^m \mid |G|$, then show that G has a subgroup of order p^m , where p is prime and $m \geq 1$.
7. Let X be an open convex subset of \mathbb{R}^n and $f, g : X \rightarrow \mathbb{R}$ be differentiable on X . If f is convex and g is positive and affine on X , then show that $\frac{f}{g}$ is pseudo convex on X .

8. Show that the n -sphere S^n is an n -dimensional differentiable manifold.
9. Prove that the zeros of

$f(z) = f(x + iy) = u(x, y) + i v(x, y)$, $x, y \in \mathbb{R}$, $i = \sqrt{-1}$ are the intersections of the curves $u(x, y) = 0$ and $v(x, y) = 0$. Also show that at a double zero of $f(z)$ each of the curves $u(x, y) = 0$, $v(x, y) = 0$ has a double point and the two curves intersect at an angle $\pi/4$.

10. Let Ω be the set of all invertible linear operators on \mathbb{R}^n onto itself. If $T \in \Omega$, $S \in L(\mathbb{R}^n, \mathbb{R}^n)$ and $\|S - T\| \|T^{-1}\| < 1$, then prove that $S \in \Omega$ and Ω is an open subset of $L(\mathbb{R}^n, \mathbb{R}^n)$. Also prove that the mapping $T \rightarrow T^{-1}$ is continuous on Ω .

Question No.

Page for Short Answer

Question No.

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895/Mathematics (Science)

ROUGH WORK

रफ़ कार्य

अभ्यर्थियों के लिए निर्देश

(इस पुस्तिका के प्रथम आवरण पृष्ठ पर तथा उत्तर-पत्र के दोनों पृष्ठों पर केवल नीली-काली बाल-प्वाइंट पेन से ही लिखें)

1. प्रश्न पुस्तिका मिलने के 30 मिनट के अन्दर ही देख लें कि प्रश्नपत्र में सभी पृष्ठ मौजूद हैं और कोई प्रश्न छूटा नहीं है। पुस्तिका दोषयुक्त पाये जाने पर इसकी सूचना तत्काल कक्ष-निरीक्षक को देकर सम्पूर्ण प्रश्नपत्र की दूसरी पुस्तिका प्राप्त कर लें।
2. परीक्षा भवन में लिफाफा रहित प्रवेश-पत्र के अतिरिक्त, लिखा या सादा कोई भी खुला कागज साथ में न लायें।
3. उत्तर-पत्र अलग से दिया गया है। इसे न तो मोड़ें और न ही विकृत करें। दूसरा उत्तर-पत्र नहीं दिया जायेगा। केवल उत्तर-पत्र का ही मूल्यांकन किया जायेगा।
4. अपना अनुक्रमांक तथा उत्तर-पत्र का क्रमांक प्रथम आवरण-पृष्ठ पर पेन से निर्धारित स्थान पर लिखें।
5. उत्तर-पत्र के प्रथम पृष्ठ पर पेन से अपना अनुक्रमांक निर्धारित स्थान पर लिखें तथा नीचे दिये वृत्तों को गाढ़ा कर दें। जहाँ-जहाँ आवश्यक हो वहाँ प्रश्न-पुस्तिका का क्रमांक तथा सेट का नम्बर उचित स्थानों पर लिखें।
6. ओ० एम० आर० पत्र पर अनुक्रमांक संख्या, प्रश्नपुस्तिका संख्या व सेट संख्या (यदि कोई हो) तथा प्रश्नपुस्तिका पर अनुक्रमांक और ओ० एम० आर० पत्र संख्या की प्रविष्टियों में उपरिलेखन की अनुमति नहीं है।
7. उपर्युक्त प्रविष्टियों में कोई भी परिवर्तन कक्ष निरीक्षक द्वारा प्रमाणित होना चाहिये अन्यथा यह एक अनुचित साधन का प्रयोग माना जायेगा।
8. प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार वैकल्पिक उत्तर दिये गये हैं। प्रत्येक प्रश्न के वैकल्पिक उत्तर के लिए आपको उत्तर-पत्र की सम्बन्धित पंक्ति के सामने दिये गये वृत्त को उत्तर-पत्र के प्रथम पृष्ठ पर दिये गये निर्देशों के अनुसार पेन से गाढ़ा करना है।
9. प्रत्येक प्रश्न के उत्तर के लिए केवल एक ही वृत्त को गाढ़ा करें। एक से अधिक वृत्तों को गाढ़ा करने पर अथवा एक वृत्त को अपूर्ण भरने पर वह उत्तर गलत माना जायेगा।
10. ध्यान दें कि एक बार स्याही द्वारा अंकित उत्तर बदला नहीं जा सकता है। यदि आप किसी प्रश्न का उत्तर नहीं देना चाहते हैं, तो संबंधित पंक्ति के सामने दिये गये सभी वृत्तों को खाली छोड़ दें। ऐसे प्रश्नों पर शून्य अंक दिये जायेंगे।
11. रफ कार्य के लिए प्रश्न-पुस्तिका के मुखपृष्ठ के अंदर वाला पृष्ठ तथा उत्तर-पुस्तिका के अंतिम पृष्ठ का प्रयोग करें।
12. परीक्षा के उपरान्त केवल ओ एम आर उत्तर-पत्र परीक्षा भवन में जमा कर दें।
13. परीक्षा समाप्त होने से पहले परीक्षा भवन से बाहर जाने की अनुमति नहीं होगी।
14. यदि कोई अभ्यर्थी परीक्षा में अनुचित साधनों का प्रयोग करता है, तो वह विश्वविद्यालय द्वारा निर्धारित दंड का/की, भागी होगा/होगी।