2016.

Set No. : 1

Question Booklet No.

RET/16/TEST-B

988 | Mathematical Science

(To be filled up by the candidate by blue/black ball point pen)

Roll No. | | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date .....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 30 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fesh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. ***A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.***
7. ***Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).***
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. ***Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.***
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 40

38.

SEAL

ROUGH WORK

रफ़ कार्य

Research Entrance Test-2016

No. of Questions : 50

Time : 2 Hours ***Full Marks : 200***

Note: **(1)** This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

(4) For mathematical Science Students only :

(i) This paper contains **three** sections :

(A) **Mathematical Section** (Q. No. **11-40** & Short Answer Questions)

(B) **Statistics Section** (Q. N0. **41-70** & Short Answer Questions)

(C) **Computer Science Section** (Q. No. **71-100** & Short Answer Questions) **A** candidate has to attempt only **one** section.

(ii) Q. No. **1 to 10** are compulsory to **all**.

(A) Mathematics Section

01. Which is not true for reactions by the S_N2 mechanism ?

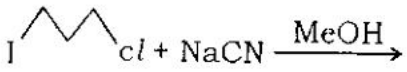
(1) proceeds through a backside attack and results in inversion

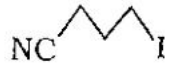
(2) tends to proceed with weak nucleophiles solvents like CH_3OH, H_2O, CH_3CH_2OH.

(3) rate of reaction proceeds from primary (fastest) > secondary >> tertiary (slowest)

(4) occurs in one step

02. Which is the main product of the following reaction ?



(1) 

(2) 

(3) 

(4) 

03. Which of the following conditions is necessary for a reaction to be spontaneous ?

(1) $\Delta S_{sur} > 0$

(2) $\Delta S_{sys} > 0$

(3) $\Delta S_{sur} + \Delta S_{sys} > 0$

(4) $\Delta S_{sur} + \Delta S_{sys} < 0$

04. Dead organs are generally stored in formalin. Formalin is :

(1) aqueous formaldehyde

(2) aqueous ferrous sulphate

(3) aqueous formic acid

(4) aqueous ferric alum

05. Regarding "carbon credits", which one of the following statement is **not** correct :

(1) The carbon credit system was ratified in conjunction with the Kyoto Protocol.

(2) Carbon credits are awarded to countires or groups that have reduced greenhouse gases below their emission quota.

(3) The goal of the carbon credit system is to limit the increase of carbon dioxide emission.

(4) Carbon credits are traded at a price fixed from time to time by the United Nations Environment Progrmme.

06. Ball bearings are used in bicycles, cars, etc., because :

(1) the actual area of contact between the wheel and axle is increased.

(2) the effective area of contact between the wheel and axle is increased

(3) the effective area of contact between the wheel and axle is reduced

(4) the actual area of contact between the wheel and axle is reduced.

07. During respiration, energy is released. It is stored in the form of :

(1) ADP (2) ATP (3) NADP (4) APP

08. Which of the following is known as Royal disease :

(1) Sickle cell anemia (2) Haemophilia

(3) Alzheimers disease (4) Colour blindness

09. The xylem in plants is responsible for :

(1) transport of water (2) transport of food

(3) transport of oxygen (4) transport of amino acids

10. Two wires, of the same material, have their lenghts in the ratio 1:2 and their diameters in the ratio 2:1. If both are stretched separately by equal weights, the ratio of increase in their lengths, $L_1 : L_2$ would be :

(1) 1:2 (2) 2:1 (3) 1:8 (4) 8:1

11. The dimension of the vector space $V = \{A = (a_{ij})_{n \times n};\ a_{ij} \in C,\ a_{ij} = -\overline{a_{ji}}\}$ over field $\mathbb{R}$ is :

(1) n^2 (2) $n^2 - 1$ (3) $n^2 - n$ (4) $n^2/2$

12. Let f is differentiable and $f(x) \neq 1$ for any real x then :

(1) f is monotonic (2) f has unique fixed point

(3) f has atmost one fixed point (4) f has no fixed point

13. Choose the incorrect, for the Initial Value Problem (IVP) :

$\frac{dy}{dx} = \frac{(y - x)}{x}$ with $y(x_0) = y_0$ has :

(1) No solutionif $y_0 = 0$, $x_0 = 0$.

(2) Unique solution if $y_0 = 1$, $x_0 = 1$

(3) Unique solution if $y_0 = 1$, $x_0 = 0$

(4) Infinite solution if $y_0 = 1$, $x_0 = 0$

14. If $(x - 1)^2 u_{xx} - (y - 2)^2 u_{yy} + 2xu_y + 2xyu = 0$ is parabolic in $S \subseteq R^2$ but not in $R^2 - \{S\}$ then S is :

(1) $\{(x, y) \in R^2; x = 1 \text{ or } y = 2\}$ (2) $\{(x, y) \in R^2; x = 1 \text{ and } y = 2\}$

(3) $\{(x, y) \in R^2; x = 1\}$ (4) $\{(x, y) \in R^2; x = 2\}$

15. The possible value of α for which the variational problem $I(y(x) = \int_0^1 (3y^2 + 2x^3 y^1)\, dx,\ y(\alpha) = 1$ has extremals are :

(1) $-1, 0$ (2) $0, 1$ (3) $-1, 1$ (4) $0, -1$

16. If a holomonic system defined by n-generalized coordinates $q_1, q_2, \ldots\ldots, q_n$ moves under the influence of conservative system, then Hamilton's equation of motion is :

(1) $\dot{p}_j = -\frac{\partial H}{\partial q_j}, \dot{q}_j = \frac{\partial H}{\partial p_j}$ (2) $\dot{p}_j = -\frac{\partial H}{\partial q_j}, \dot{q}_j = -\frac{\partial H}{\partial p_j}$

(3) $\dot{p}_j = \frac{\partial H}{\partial p_j}, \dot{q}_j = \frac{\partial H}{\partial q_j}$ (4) $\dot{p}_j = \frac{\partial H}{\partial p_j}, q_j = -\frac{\partial H}{\partial q_j}$

17. Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1$, $n = 1, 2, \ldots\ldots$ If $\int_{-1}^{1} \sum_{j=1}^{n} j(2_j+1)P_j(x)^2 dx = 20$ n is e qual to :

(1) 2 (2) 3 (3) 4 (4) 5

18. Consider the topology $\tau = \{G \subseteq R : R\backslash G\}$ is compact in $(R, \tau_U) \cup \{\phi, R\}$ on where is the usual topology on R and ϕ is the empty set. Then (R, τ) is :

(1) A connected Hausdorff space

(2) Connected but not Hausdorff

(3) Hausdorff but not connected

(4) Neither connected nor Hausdorff

19. The geometrical representation of a complex number divided by 'i' is :

(1) Reflection abot the x-axis

(2) Reflection abot the y-axis

(3) Rotation throgh 90^0 about the origin in the counter clockwise direction

(4) Rotation through 90^0 about the origin in the clockwise direction

20. The truncation error of the following method $u_{j+1} = u_j + \frac{1}{6}(k_1 + 2k_2 + 2K_3 + k_4)$ is of order :

(1) 3 (2) 4 (3) 5 (4) 6

21. The sum of residues of the function $f(t) = \frac{z^2}{(z-1)(z-2)(z-3)}$ at $z = 1, 2, 3$ and ∞ is :

(1) 1/2 (2) 1 (3) -8 (4) 0

22. If $f : R \to R$ is continous then which one of the following cannot be the image of [0, 1] under f :

(1) {0} (2) (0, 1) (3) [0, 1) (4) [0,1]

23. The value of the integral of $\frac{1}{z}$ along the semi circular arc $|z| = 1$ form $z = -1$ to $z = +1$, lies below the real axis, is :

(1) pi (2) - pi (3) 0 (4) 2 pi

24. If $A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ then the value of e^A is :

(1) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} e^2$ (2) $\begin{pmatrix} e^2 & e^3 \\ 1 & e^2 \end{pmatrix}$

(3) $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} e^2$ (4) $\begin{pmatrix} e & e^3 \\ 1 & e \end{pmatrix}$

25. The maximum number of independent components of tensor of rank three, symmetry in last two indices in V_n, are :

(1) $\frac{n(n+1)}{2}$ (2) $\frac{n^2(n+1)}{2}$

(3) $\frac{n(n-1)}{2}$ (4) $\frac{n^2(n-1)}{2}$

26. The solution of the differential equation $\frac{d^3y}{dx^3} - 2y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = e^{5x}$ is :

(1) $y = Ae^x + Be^{-x} + Ce^{2x} + \frac{e^{5x}}{72}$ (2) $y = Ae^{-x} + Be^x + Ce^{2x} + \frac{e^{5x}}{72}$

(3) $y = Ae^x + Be^{-x} + Ce^{2x} + \frac{xe^{5x}}{72}$ (4) $y = Ae^x + Be^{-x} + Ce^{2x} - \frac{xe^{5x}}{72}$

27. The locus of complex number, z, satisfying $|z - 2| + |z + 2| = 6$ is :

(1) Line segment (2) Circle

(3) Ellipse (4) Straight Line

28. For a continuous function $f : R \to R$, let $Z(f) = \{x \in R : f(x) = 0\}$. Then Z (f) is always :

(1) compact (2) open

(3) connected (4) closed

29. Let f(x) = max (sinx, cosx), for all $x \in R$. Then :

(1) f is differentiable on R

(2) f is differentiable on R except at 0

(3) f is nowhere differentiable

(4) f is differentiable except at a countable set of points

30. For the metric $ds^2 = -a(dx^1)^2 + b(dx^2)^2 + c(dx^3)^2 + d(dx^4)^2$, where a,b,c,d are functions of x^1 (i = 1, 2, 3, 4), the value of the Christoffel symbol [2,11] is :

(1) $\frac{1}{2}\frac{\partial a}{\partial x^2}$ (2) $-\frac{1}{2}\frac{\partial a}{\partial x^2}$

(3) $\frac{1}{2}\frac{\partial b}{\partial x^1}$ (4) $-\frac{1}{2}\frac{\partial b}{\partial x^2}$

31. The function $f(x, y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$ is :

(1) continuous at (0,0) but its partial derivatives f_x and f_y do not exist at (0,0)

(2) discontinuous at (0,0) but f_x and f_y exist at (0,0)

(3) continuous at (0,0) and f_x exists but f_y does not exist at (0,0)

(4) continuous at (0,0) and f_y exists but f_x does not exist at (0,0)

32. The boundary of $\{r + is : r, s \in Q\}$ in the topological space C (under usual topology) is :

(1) the empty set (2) $\{x + iy : x, y \in R - Q\}$

(3) $\{is : s \in R - Q\}$ (4) C

33. For a homogeneous medium containing charges and currents, the value of $\nabla^2 E - \mu\epsilon\frac{\partial^2 E}{\partial t^2}$ is :

(1) $\epsilon\nabla p + \mu\frac{\partial}{\partial t}J$ (2) $\frac{1}{\epsilon}\nabla p + \mu\frac{\partial}{\partial t}J$

(3) $\epsilon\nabla p + \frac{1}{\mu}\frac{\partial}{\partial t}J$ (4) $\frac{1}{\epsilon}\nabla p + \frac{1}{\mu}\frac{\partial}{\partial t}J$

34. The continuous function $f:R \to R$ defined by $f(x) = (1 + x^2)^{2016}$ is :

(1) onto but not one-one (2) one-one but not onto

(3) both one-one and onto (4) neither one-one nor onto

35. The number of elements of order 5 in $Z_{25} \oplus Z_5$ are :

(1) 24 (2) 25 (3) 30 (4) 100

36. Consider the following LPP min $z = 10x_1 + x_2 + 5x_3$ such that $5x_1 - 7x_2 + 3x_3 \geq 50$, $x_1, x_2, x_3 \geq 0$, then its optimal value is :

(1) 50/3 (2) 250/3 (3) 10/3 (4) 100/3

37. The value of cos θ for the angle θ between $f(t) = 2t - 1$ and $g(t) = t^2$ in the vector space V of polynomial with inner product $\{f,g\} = \int_0^1 f(t)\, g(t)\, dt$ is :

(1) $\frac{\sqrt{15}}{4}$ (2) $\frac{\sqrt{15}}{6}$ (3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{\sqrt{6}}$

38. The Fourier transform of $f(x) = \begin{cases} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a \end{cases}$ is :

(1) $\sqrt{\frac{\pi}{2}} \frac{\text{Sin sa}}{s}$ (2) $\sqrt{\frac{\pi}{2}} \frac{\text{Cos sa}}{s}$

(3) $\sqrt{\frac{2}{\pi}} \frac{\text{Sin sa}}{s}$ (4) $\sqrt{\frac{2}{\pi}} \frac{\text{Cos sa}}{s}$

39. The value of integral $\int_1^4 [\log_e x]\, dx$ where [ ] denotes greatest integer function is :

(1) log 4 (2) 1/2 (3) 4 + e (4) 4 - e

40. Solution of the Poission's equation $u_{xx} + u_{yy} = -1$ in the square $|x| \leq 1$, $|y| \leq 1$ with $u = 0$ at $x = \pm 1$ and $y = \pm 1$ is :

(1) $u(x, y) = \frac{1}{16}(1 - x^2)(1 - y^2)$

(2) $u(x, y) = \frac{5}{16}(1 - x^2)(1 - y^2)$

(3) $u(x, y) = \frac{5}{16}(1 + x^2)(1 + y^2)$

(4) $u(x, y) = \frac{1}{16}(1 + x^2)(1 + y^2)$

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

01. Define gamma function on complex plane and show that it has only simple poles as singularities.

02. Find the nature of extremals of the functional $\int_1^2 \frac{\left[1+y'^2\right]^{\frac{1}{2}}}{x}$ dx under the boundary condition y(1) = 0, y(2) = 2.

03. Define a regular surface. Show that graph of a smooth function is a regular surface.

04. Show that a necessary and sufficienct condition for a curve to be a spherical is that its radius of curvature p and radius torsion satisfy $\frac{p}{\sigma}+\frac{d}{ds}(p'\sigma) = 0$ where s is arc lenght parameter and $p' = \frac{dp}{ds}$

05. State and prove Euler formula for normal curvature at point on a regular surface.

06. Show that the equation $z^5 + 15z + 1 = 0$ has four roots in the annulus $\frac{2}{3} < |z| < 2$

07. Show that the age of the universe (t_0) in terms of observable parameters for open universe is

$$t_0 = \frac{q_0}{H_0(1-2q_0)^{2/3}} + \left[\frac{\sqrt{1-2q_0}}{q_0} - \cosh^{-1}\left(\frac{1-q_0}{q_0}\right)\right]$$; where H_0 and q_0 are the Hubble constant and deceleration parameter at present.

08. Evaluate the intergral $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's one-third rule taking k= 8. The arguments and corresponding values of the function f(x) are tabulate below :

x	0.000	0.125	0.250	0.375	0.500	0.625	0.875	1.000
f(x)	1.00000	0.98461	0.94117	0.87671	0.80000	0.64000	0.56637	0.50000

09. Show that the transformation $w = z^2$ of complex plane transform circle centred on real line in z-plane to limacon's in w-plane.

10. If $f_n : R \to R$ be the function $fn(x) = \frac{1}{n^3\left(x-\frac{1}{n}\right)^2+1}$ and $f : R \to R$ be the zero function then show that $f_n(x) \to f(x)$ for each $x \in R$ but f_n does not converge uniformly to f.

(B) Statistics Section

41. Which of following is not a correct statement about Bivariate Normal Distribution ?

(1) Marginaldistributions are normal

(2) Both variables are linearly related

(3) Conditional variance is constant

(4) Conditional variance is not constant

42. A sample of size n($\geq$2) is drawn without replacement from a finite population os size N, using an arbitrary sampling scheme. Let πi denote the inclusion probability of the i-th unit and πij, the joint probability of units i and , $1 \leq i < j \leq N$. Which of the following statement is always true ?

(1) $\sum_{i=1}^{N} \pi i = n$

(2) $\sum_{i=1}^{N} \pi ij = n\pi i,\ 1 \leq i \leq N$

(3) $\pi ij > 0$ for all I, j, $1 \leq i < j \leq N$

(4) $\pi i \pi j - \pi ij > 0$ for all I, j, $1 \leq i < j \leq N$

43. Hundred (100) tickets are marked 1,2, ... 100 and are arranged at random. Four tickets are picked from these tickets and are gtiven to four persons A,B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A, B, C, D) ?

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{12}$

44. Suppose $X_1, X_2, \ldots\ldots X_n$ is a random sample from U $(0, \theta)$, > 0. Let $X_{(1)} \leq X_{(2)} \leq \ldots\ldots \leq X_{(n)}$ be the order statistics and S be sample mean. Consider the two unbiasedestimators for θ : T1 = 2S and T2 = $\left(\frac{n+1}{n}\right) X_{(n)}$ Then $\lim_{n \to \infty} \frac{VAR(T2)}{VAR(T1)}$ =

(1) 0 (2) 1 (3) ∞ (4) 12

45. Let $X_1, X_2, \ldots\ldots X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement.

(1) $(X_1 - \mu)' \Sigma^{-1} (X_1 - \mu)$ has chi-square distribution with 1 d.f.

(2) $\sum_{i=1}^{n} (Xi - \mu)(Xi - \mu)$ has Wishart distribution with n d. f.

(3) SS' (S is sample mean vector) has Wishart distribution with pd.f.

(4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed

46. A factorial experiment invlving four factors F1, F2, F3, and F4 each at 2 levels, 0 and 1, is planned in 4 blocks each of size 4. One of these block has the following contents :

F1	F2	F3	F4
0	0	0	0
0	0	0	0
1	0	1	1
1	1	1	0

The confounded factorial effect are :

(1) F1F2, F1F3, F2F3

(2) F1F3, F1F2F4, F2F3F4

(3) F3F4, F1F2F3, F1F2F4

(4) F1F4, F2F3, F1F2F3F4

47. Suppose the comulative distribution function of failure time T of a component is :

$1 - \exp(-ct^{\alpha})$, $t > 0$, $\alpha > 1$, $c > 0$

Then the hazard rate of $\lambda(t)$ is :

(1) Constant

(2) Monotone increasing in t

(3) Monotone decreasing in t

(4) Non-monotone function in t

48. Consider the following probability mass function P(x) where the parameters (θ_1, θ_2) take values in parameter space

$$\left\{\left(\frac{1}{3},3\right),\left(\frac{1}{2},2,\right),\left(2,\frac{1}{2}\right),\left(3,\frac{1}{3}\right)\right\}$$

$(?_1,?_2)$ x	$\left(\frac{1}{3},3\right)$	$\left(\frac{1}{2},2,\right)$	$\left(2,\frac{1}{2}\right)$	$\left(3,\frac{1}{3}\right)$
1	1/11	1/7	1/8	1/9
2	1/11	1/14	1/16	1/9
3	8/11	5/7	3/4	2/3
4	1/11	1/14	1/16	1/9

Let X be a random observation from this distribution. If the observed value of X is 3, then

(1) MLE of $\theta_1 = 1/3$, MLE of $\theta_2 = 3$

(2) MLE of $\theta_1 = 1/2$, MLE of $\theta_2 = 2$

(3) MLE of $\theta_1 = 2$, MLE of $\theta_2 = 1/2$

(4) MLE of $\theta_1 = 3$, MLE of $\theta_2 = 1/3$

49. A symmetric die is thrown 720 times. The lower bound for the probability of getting 100 to 140 sixes is :

(1) 0.25 (2) 0.50 (3) 0.75 (4) 0.05

50. 10 If X_1 follows B (n_1, p_1) and X_2 follows B(n_2, p_2) independently of X_1 then $Z = x_1 + x_2$ will follow :

(1) Binomial distribution if $n_1 = n_2$.

(2) Binomial distribution if $p_1 = p_2$.

(3) Poisson distribution if $n_1 = n_2$.

(4) Poisson distribution if $p_1 = p_2$.

51. Suppose X has a geometrice distribution with parameter p :

$P(X = t) = q^t p$; t = 0, 1, 2 .....

Which of the following is correct :

(1) $P(y = t/X > k) = pq^t$ (2) $P(y = t/X < k) = pq^t$

(3) $P(y = t/X \geq k) = pq^t$ (4) P(y =t/X k) = pq^t

52. let X has the M.G.F. as : $M(t) = \frac{\left(1+2e^t\right)^3}{27}$

The mean and variance of X are

(1) 1/2 and 9/4 (2) 2 and 4/9

(3) 2 and 2/3 (4) 1 and 2/3

53. If $X \sim N(0, 1)$ then $Z = \frac{1}{2} X^2$ follows :

(1) Chi-square with 1 d.f. (2) Gamma $\left(\frac{1}{2}\right)$

(3) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ of first kind (4) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ of secondt kind

54. Which of the following statements is correct ?

(1) Convergence in Probability is stronger than the almost sure convergent.

(2) Convergence in Probability is stronger than the Convergence in law.

(3) Convergence in law is stronger than the Convergence in Probability.

(4) Convergence in law is stronger than the almost sure convergent.

55. Let $X_1, X_2, \ldots\ldots, X_n$ are iid variables from N $(?, ?^2)$, then the statistics $\sum Xi \sum Xi^2$ is :

(1) Jointly sufficient and complete

(2) Sufficient but not complete

(3) Complete but not sufficient

(4) Neither sufficient nor complete

56. A sequence $\{A_n\}$ defined as :

$$A_n\left\{\omega \text{ s. t. } 0 < \omega < b + \frac{(-1)^n}{n}\right\}; (b > 1)$$

(1) Monotone non-decreasing.

(2) Monotone non-increasing.

(3) Non-monotone but convergent.

(4) Non-monotone and non-convergent.

57. Which of the following is/are correct ?

(i) Every monotone field is a sigma field

(ii) Every field containing infinite elements is always a sigma field

(1) (i) and (ii) both are correct

(2) (i) and (ii) both are incorrect

(3) (i) is correct but (ii) is incorrect

(4) (i) is incorrect but (ii) is correct

58. Consider the following assertions :

(i) A minimal sufficient statistic is always complete.

(ii) A minimal sufficient statistic may not be complete.

(iii) A complete sufficient statistic is always minimal sufficient.

(iv) A complete sufficient statistic may not be minimal sufficient.

(1) Only (i) is true

(2) Only (ii) and (iv) are true

(3) Only (ii) and (iii) are true

(4) Only (i) and (iii) are true

59. Suppose $X \sim n(0, \sigma^2)$; then $Y = X^2$ will follow :

(1) Pareto distribution

(2) Cauchy distribution

(3) Rayleigh distribution

(4) Student's t- distribution

60. A Bayes decision rule :

(1) is never admissible

(2) is always admissible

(3) admissible only if it is unique

(4) admissible if it is unique except under risk equivalence

61. What is the effect of an outlier on the value of a correlation coefficeint ?

(1) An outlier willalways decrease a correlation coefficient.

(2) An outlier willalways increase a correlation coefficient.

(3) An outlier might either decrease or increase a correlation coefficient, depending on where it is in relation to the other points

(4) An outlier will have no effect on a correlation coefficient

62. The lifetime of a bulb is exponentially distributed with mean 100 hours. The bulb remains switched on for exactly 4 hours everyday and remains switched off the remaining time. What is the probability that the bulb stops working on or before the 25th day ?

(1) $\frac{1-e^{-1}}{1-e^{\frac{1}{25}}}$ (2) $1-e^{\frac{1}{25}}$

(3) $1 - e^{-1}$ (4) e^{-1}

63. A Markow chain is said to be irreducible if :

(1) Possible to get from any where to any where

(2) Chain has probability one of returning to the initial state

(3) Chain has probability zero of returning to the initial state

(4) None of the above statement is correct.

64. In acceptance-rejection method a function t(x) that majorizes the density function f(x) for all x is :

(1) A density function

(2) A density function only for some x

(3) A density function only for values less than x

(4) Not necessarily a density function

65. The samples from a two-parameter Weibull distribution for some known values of shape and scale parameters can be generated with the help of inverse transfrom method by using the transformation :

(1) $\{-\text{scale} \log \{1 - u\}\}^{\text{shape}}$ (2) $\{1 - \exp(-x^{\text{shape}}/\text{scale})\}u$

(3) $\{-\text{scale} \log \{1 - u\}\}^{1/\text{shape}}$ (4) $\{\exp(-x^{\text{shape}}/\text{scale})\}u$

66. Suppose you have a posterior distribution $p(\theta/x)$ based a sample of size n i. e., $x : x_1, x_2 \ldots\ldots, x_n \sim f(x/\theta)$ and a prior distribution $g(\theta)$. If an additional observation x_{n+1} is added at a later stage, then the updated posterior will be :

(1) Same as $p(\theta/x)$

(2) Proportional to $f(x_{n+1})/\theta)\, p(\theta/x)$

(3) Proportional to $\{f(x/\theta) + f(x_{n+1})/\theta)\, p(\theta/x)$

(4) Cannot be updated

67. Consider an independent Bernoulli trails with success probability θ Using the prior proportional to $\theta^{a-1}(1-\theta)^{b-1}$. What is the Bayes estimator of θ when the loss is squared error loss ?

(1) $\frac{a+t}{a+b+n}$ (2) $\frac{a-t}{a+b+n}$

(3) $\frac{a+t}{n}$ (4) $\left(\frac{a+t}{n}\right)^2$

Where is the number of success in n independent trials.

68. For a Likelihood ratio test, let us define $\lambda(x) = \frac{\sup f_{\theta \in \Theta_0}(x/0)}{\sup f_{\theta \in \Theta}(x/0)}, \Theta = \Theta_0 \cup \Theta_1$. Then assuming some regularity conditions, which statement about the quantity $k = -2\log \lambda(x)$ is true ?

(1) k follows chi-square distribution with d.f. equals to the number of independent parameters in Θ_0

(2) k follows chi-square distribution with d.f. equals to the difference between the number of independent parameters in Θ and the number in Θ_0

(3) k follows f distribution with n_1 and n_2 degrees of freedom where n_1 and n_2 are the number of parameters in Θ_0 and Θ respectively.

(4) k follows chi-square distribution with d.f. equals to the number of independent parameters in Θ

69. WHO recommended method of sampling to estimate immunization coverage in developing countrie is :

(1) Simple random sampling

(2) Stratified random sampling

(3) 30 cluster random sampling

(4) Muli-stage random sampling

70. Calculation of relative risk requires a :

(1) Prospective study (2) Retrospective study

(3) Cross sectional study (4) Exploratory study

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. What is complete and partial confounding ?

2. Explain the basic principle on which quadrature formulae are developed.

3. Explain by means of an example the conditional transfer in R or any high level language.

4. What is data warehousing ?

5. State and prove Rao-Blackwell theorem.

6. Define odds ratio in case-control study design by means of an example.

7. Define Jeffreys prior and work out Jeffreys prior for success probability p in Bernoulli distribution.

8. What do you mean by stochastic simulation ?

9. What is robust regression ?

10. Define Cox's proportional hazards model for several covariates.

(C) Computer Science Section

71. Consider the following segment of C program :

```
int x, y, n;
x = 1;
y = 1;
If (n > 0)
x = x + 1 ;
else
y = y - 1 ;
```

After execution of above program segment the value of x and y if n = 1 is :

(1) x = 2, y = 0; (2) x = 1, y = 0;

(3) x = 1, y = 1; (4) x = 2, y = 1;

72. Consider the following segment of C program :

```
int i, j:
j = 0;
for (i =0; i < =  5; i = i + 2/3)
{
j = j + 1 ;
}
```

The number of times the body of for loop is executd :

(1) 9 (2) 8

(3) infinite (4) 11

73. How many of the followig declarations are correct ?

```
Int x ;
float letter, DIGIT ;
double = p, q
m, n, z : INTEGER
long int m; count;
long float temp;
```

(1) Three are correct (2) One is correct
(3) Two are correct (4) All six ar incorrect

74. Consider the following C program :

```
main ( )
{
int num 1. num 2;
scanf ("%2d%5d", & num 1, & num 2);
print f(:%d%d", num 1, num 2);
}
```

If the data input to the program 31426 and 50, then the output will be :

(1) 31426,50 (2) 50,31426
(3) 31,426 (4) 3142,650

75. Consider the following ANSI union :

```
union item
{
int m;
float x;
char c;
};
```

Total memory location required to store any union variable of type item is :

(1) 2 byte (2) 4 byte
(3) 6 byte (4) 7 byte

76. When simplified with Boolean Algebra, the expression $(x + y)(x + z)$ simplifies to :

(1) x (2) $x + x(y + z)$

(3) $x(1 + yz)$ (4) $x + yz$

77. How many 1's are present in the binary representation of $3 \times 512 + 7 \times 64 + 5 \times 8 + 3$:

(1) 8 (2) 9 (3) 10 (4) 11

78. The decimal number equivalent of $(4057.06)_8$ is :

(1) 2095, 75 (2) 2095, 075

(3) 2095, 937 (4) 2095, 0937

79. $(734)_8 = (\ )_{16}$:

(1) CID (2) DCI (3) ICD (4) IDC

80. Two fuzzy sets A and B are given with membership :

$\mu_A(x) = \{0.2, 0.4, 0.8, 0.5, 0.1\}$

$\mu_B(x) = \{0.1, 0.3, 0.6, 0.3, 0.2\}$

Then the value of $\mu_{A \cup B}$ will be :

(1) {0.9, 0.7, 0.4, 0.8, 0.9} (2) {0.2, 0.4, 0.8, 0.5, 0.2}

(3) {0.3, 0.7, 1.4, 0.8, 0.3} (4) {0.1, 0.3, 0.6, 0.3, 0.1}

81. If a pixel of an image is shuffled, then he parameter that not change is :

(1) Histogram, Mean, Covariance

(2) Histogram, Mean, Entropy

(3) Histogram, Covariance

(4) Covariance, Entropy

82. The information is accessed in stack data structure is :

(1) LIFO (2) FIFO

(3) Random (4) LIFO and Random Both

83. The preorder traversal of abinary tree is DEBFCA. The root node of binary tree is :

(1) B (2) C (3) A (4) D

84. Chromatic number of bipartite graph is :

(1) 4 (2) 2 (3) 3 (4) 1

85. Which of the following pair (A,B) of events is mutually exculusive, in the random experiment of tossing of a coin four times ?

(1) A : Obtaining at least three heads, B : Obtaining at least three tails.

(2) A : Obtaining at least two heads, B : Obtaining at most two tails.

(3) A : Obtaining at least two heads, B : Obtaining at least two tails.

(4) A : Obtaining at least three heads, B : Obtaining at most three tails.

86. Assuming the normal distribution, suppose that a 95% confidence interval for mean μ is (50, 60). Which of the following could possibly be a 99% confidence interval for the same ?

(1) (52, 58) (2) (52, 62) (3) (48, 58) (4) (48, 62)

87. Let T(n) be the function defined by T(0) = 1 and T(n) = T(n - 1) + n, n > = 1. Which of the following is true :

(1) $T(n) = O(n^2)$ (2) $T(n) = O(\sqrt{n})$

(3) $T(n) = O(\log_2 n)$ (4) $T(n) = O(n)$

88. In AVL tree, difference between height of left and right subtrees is :

(1) Less than 1 (2) Less than equal to 1

(3) equal to 1 (4) greater than equal to 1

89. The output of D-flip flop :
(1) same as input
(2) complement of input
(3) not depend on input
(4) depend on past input and clock

90. Fundamental period of $x(n) = 10 \cos(\pi n/3)$ is :
(1) 3 (2) 1/3 (3) 6 (4) $\pi n/3$

91. A and B are two logical statements. Statement B is logical equivalent to statement A iff :
(1) $(A \rightarrow B) \wedge (B \rightarrow A)$ is tautology
(2) $(A \rightarrow B) \mid\mid (B \rightarrow A)$ is tautology
(3) $A \rightarrow B$ is contradiction
(4) $A \rightarrow B$ is tautology

92. Nayquist rate of the signal $x(t) = 3 \cos(50\pi t) + 10 \sin(30\pi t) - \cos(100\pi t)$ is :
(1) 50 Hertz (2) 300 Hertz
(3) 100 Hertz (4) 150 Hertz

93. Postfix expression equivalent to infix expression (A - B)* (D/E) is :
(1) ABDE*/- (2) ABDE-/*
(3) AB - DE/* (4) None of these

94. The Laplace transform of $\sin \omega t$ is :
(1) $\frac{s^2}{s^2+\omega^2}$ (2) $\frac{\omega^2}{s^2+\omega^2}$
(3) $\frac{\omega}{s^2+\omega^2}$ (4) $\frac{\omega}{s^2+\omega}$

95. A casual signal has Z-transform with complex conjugate pole located at unit circle ($|z|=1$). The signal is :

(1) Oscillatory and decaying (2) Oscillatory and increasing

(3) Oscillating (4) Constant

96. The function $f(x) = \sin(x)$ is decreasing in :

(1) $(0, \pi)$ (2) $(0, \pi/2)$

(3) $(\pi/2, 3\pi/2)$ (4) $(\pi, 2\pi)$

97. If $f(x) = \sin 2x$, $0 \leq x \leq \pi/2$, the value x at which tangent is parallel to x-axis :

(1) $\frac{p}{4}$ (2) $\frac{p}{3}$ (3) $\frac{p}{2}$ (4) p

98. If A and B are two square matrices of same order, then which of the following is not correct ?

(1) $\det(AB) = \det(A) + \det(B)$ (2) $\det(A^T) = \det(A)$

(3) $\det(A^{-1}) = 1/\det(A)$ (4) $\det(AB) = \det(A).\ \det(B)$

99. Worst case time complexity of heap sort is :

(1) $O(n)$ (2) $O(n^2)$

(3) $O(n^3)$ (4) $O(n\log_2 n)$

100. The probability mass function of a random variable X is given below :

$f(x) = x/15$; $x = 1, 2, 3, 4, 5$

$= 0$; otherwise

Then the conditional probability that X lies between 1/2 and 5/2 given that X is greater than 1 is :

(1) 1/7 (2) 3/7 (3) 2/15 (4) 1/5

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. (a) Give data structure in which linear list is implemented using random access. Using this representation/data structure write an algorithm for inserting and deleting an element form the list.

(b) Describe stack and queue data structure. Explain PUSH and POP operation on a stack.

2. (a) Suppose the following list of letters is inserted in order into an empty Binary search tree :

J, R, D, G, T, E, M, H, P, A, F, Q

(i) Find the final Binary tree

(ii) Give sequential representation of the tree in (i)

(iii) Give the link representation of the tree in (i) using three array.

(iv) Find the inorder, preorder, and postorder traversal of Binary tree.

(v) Consider the Binary tree (i), give tree after the node M and D is deleted.

(b) A Binary tree T has 9 nobes. The inorder and preorder traversal of T yield the following sequence of nodes :

INORDER : E, A, C, K, F, H, D, B, G

PREORDER : F, A, E, K, C, D, H, G, B

Draw the Binary tree and also give its postorder traversal

3. Discuss maximum likelihood method of estimation. Obtain estimators of μ and σ^2 on the basis of random samples of size n drawn from $N(\mu, \sigma^2)$ by maximum likelihood method.

4. Define pole and zero of a Z-transform, and determine the zero and pole of the signal $x(n) = (1/2)^n u(n)$, where u(n) is the unit step signal.

5. (a) Explain how the Fourier transform is relatedtothe Z-transform.

   (b) State and explain the Dirichlet conditions.

   (c) Explain band width of signal, low frequency and high frequency signal.

6. (a) (i) How structure differ from each other

   (ii) How structure, union, and bit filed differ from each

   (b) Define a structure called **cricket** that will describe the information player name, team name, batting average. Using structure **cricket**, declare an array player with 5 elements.

   (c) In what ways does a switch statement is differ from if statement, Write a C program for marks range to grade conversion using switch and case, for following data :

Marks Range	**Grade**
< 40	E
40-54	D
55-69	C
70-85	B
> 85	A

7. Show thae $(\exists x)$ M (x) follows logically from the premises $(\forall x)$ (H(x) $\rightarrow$ M(x) and $(\exists x)$ H(x)

8. Define fuzzy set. Generalize the intersection, union operation on crisp set using maximum and minimum for fuzzy set.

9. Density function of a random variable X is defined as follows :

$$f_x(x) = \begin{cases} kx; 0 \le x \le 2 \\ 0; \text{otherwise} \end{cases}$$

Find the following :

(i) Value of k

(ii) Find the probability $P(0 < X < 0.75)$

(iii) Determine c so that $P(X \le c) = 0.81$

10. (a) Suppose the following number is stored in array A :

32, 51, 27, 85, 66, 23, 13, 57

Simulate the bubble sort procedure

(b) Supose A is stored array with 200 elements, and suppose a given element x appears with the same probability in any place in A. Find the worst-case running time f(n) the average case running time g(n) to find x in A using the Binary search.

Question No.	Page for Short Answer

Question No. Page for Short Answer

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ROUGH WORK

रफ़ कार्य

अभ्यर्थियों के लिए निर्देश

(इस पुस्तिका के प्रथम आवरण पृष्ठ पर तथा उत्तर-पत्र के दोनों पृष्ठों पर केवल नीली-काली बाल-प्वाइंट पेन से ही लिखें)

1. प्रश्न पुस्तिका मिलने के 30 मिनट के अन्दर ही देख लें कि प्रश्नपत्र में सभी पृष्ठ मौजूद हैं और कोई पृष्ठ छूटा नहीं है। पुस्तिका दोषपूर्ण पाये जाने पर इसकी सूचना तत्काल कक्ष निरीक्षक को देकर उससे प्रश्नपत्र की दूसरी पुस्तिका प्राप्त कर लें।
2. परीक्षा भवन में ***लिफाफा रहित प्रवेश-पत्र के अतिरिक्त,*** लिखा या सादा कोई भी खुला कागज साथ में न लायें।
3. ***उत्तर-पत्र अलग से दिया गया है। इसे न तो मोड़ें और न ही विकृत करें। दूसरा उत्तर-पत्र नहीं दिया जायेगा। केवल उत्तर-पत्र का ही मूल्यांकन किया जायेगा।***
4. अपना अनुक्रमांक तथा उत्तर-पत्र का क्रमांक प्रथम आवरण-पृष्ठ पर पेन से निर्धारित स्थान पर लिखें।
5. ***उत्तर-पत्र के प्रथम पृष्ठ पर पेन से अपना अनुक्रमांक निर्धारित स्थान पर लिखें तथा नीचे दिये वृत्तों को गाढ़ा कर दें। जहाँ-जहाँ आवश्यक हो वहाँ प्रश्न-पुस्तिका का क्रमांक तथा सेट का नम्बर उचित स्थानों पर लिखें।***
6. ***ओ० एम० आर० पत्र पर अनुक्रमांक संख्या, प्रश्नपुस्तिका संख्या व सेट संख्या (यदि कोई हो) तथा प्रश्नपुस्तिका पर अनुक्रमांक और ओ० एम० आर० पत्र संख्या की प्रविष्टियों में उपरिलेखन की अनुमति नहीं है।***
7. ***उपर्युक्त प्रविष्टियों में कोई भी परिवर्तन कक्ष निरीक्षक द्वारा प्रमाणित होना चाहिये अन्यथा यह एक अनुचित साधन का प्रयोग माना जायेगा।***
8. ***प्रश्न-पुस्तिका में प्रत्येक प्रश्न के चार वैकल्पिक उत्तर दिये गये हैं। प्रत्येक प्रश्न के वैकल्पिक उत्तर के लिए आपको उत्तर-पत्र की सम्बन्धित पंक्ति के सामने दिये गये वृत्त को उत्तर-पत्र के प्रथम पृष्ठ पर दिये गये निर्देशों के अनुसार पेन से गाढ़ा करना है।***
9. प्रत्येक प्रश्न के उत्तर के लिए केवल एक ही वृत्त को गाढ़ा करें। एक से अधिक वृत्तों को गाढ़ा करने पर अथवा एक वृत्त को अपूर्ण भरने पर वह उत्तर गलत माना जायेगा।
10. ***ध्यान दें कि एक बार स्याही द्वारा अंकित उत्तर बदला नहीं जा सकता है। यदि आप किसी प्रश्न का उत्तर नहीं देना चाहते हैं, तो संबंधित पंक्ति के सामने दिये गये सभी वृत्तों को खाली छोड़ दें। ऐसे प्रश्नों पर शून्य अंक दिये जायेंगे।***
11. रफ कार्य के लिए प्रश्न-पुस्तिका के मुखपृष्ठ के अंदर वाला पृष्ठ तथा उत्तर-पुस्तिका के अंतिम पृष्ठ का प्रयोग करें।
12. परीक्षा के उपरान्त *केवल* ***ओ एम आर उत्तर-पत्र*** परीक्षा भवन में जमा कर दें।
13. परीक्षा समाप्त होने से पहले परीक्षा भवन से बाहर जाने की अनुमति नहीं होगी।
14. यदि कोई अभ्यर्थी परीक्षा में अनुचित साधनों का प्रयोग करता है, तो वह विश्वविद्यालय द्वारा निर्धारित दंड का/की, भागी होगा/होगी।