

- 20) Which of the following series is divergent?
- (A) $\sum \sin \frac{1}{n}$ (B) $\sum \sin \frac{1}{n^2}$
 (C) $\frac{1}{\sqrt{n}} \tan \frac{1}{n}$ (D) none of these
- 21) The expansion of $\tan x$ in powers of x by Maclaurin's theorem is valid in the interval
- (A) $(-\infty, \infty)$ (B) $(-\frac{3\pi}{2}, \frac{3\pi}{2})$
 (C) $(-\pi, \pi)$ (D) $(-\frac{\pi}{2}, \frac{\pi}{2})$
- 22) The value of 'c' of Lagrange's mean value theorem for $f(x) = x(x - 1)$ in $[1, 2]$ is given by
- (A) $5/4$ (B) $3/2$
 (C) $7/4$ (D) $11/6$
- 23) The integral $\int_0^3 [x] dx$ (where $[x]$ is greatest integer function) is
- { (A) 0 (B) 1 (C) 2 (D) 3 }
- 24) The series $\sum n^{-x}$ is
- (A) uniformly convergent in $[1 + \delta, \infty)$, $\delta \geq 0$
 (B) uniformly convergent in $[1 + \delta, \infty)$, $\delta > 0$
 (C) uniformly convergent in $[\delta, \infty)$, $\delta \geq 0$
 (D) uniformly convergent in $[\delta, \infty)$, $\delta > 0$
- 25) If (X, ρ) is metric space, then for all $x, y \in X$
- (A) $\rho(x, y) \leq 0$ (B) $\rho(x, y) = 0$ for some $x \neq y$
 (C) $\rho(x, y) = 0$ if $x = y$ (D) none of these
- 26) The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{z^n}{n}$ is
- (A) 1 (B) 0 (C) $\sqrt{3}$ (D) $\sqrt{5}$
- 27) If $f(z) = z|z|^2$, then $f(z)$ is differentiable
- (A) at all points z (B) at $z = 1$
 (C) only for $z = 0$ (D) at none of these points
- 28) The function $w = \cos z$ is
- (A) unbounded and entire (B) bounded and entire
 (C) bounded but nowhere analytic (D) unbounded but nowhere analytic
- 29) The integral $\int_{|z+i\pi|=1} \frac{z}{e^z - 1} dz$ has the value
- (A) zero (B) πi (C) 2π (D) none of these
- 30) For $\frac{d^2y}{dx^2} + 4y = \tan 2x$ solving by variation of parameters. The value of Wronskian W is
- (A) 1 (B) 2 (C) 3 (D) 4

31) Let X have the probability density function

$$f(x) = \begin{cases} 0.75(1-x^2), & x \in [-1,1] \\ 0, & \text{otherwise} \end{cases}$$

The probability distribution function $F(x)$ is given by

- (A) $F(x) = 0.5 + 0.75x - 0.25x^3, x \in (-1,1]$
- (B) $F(x) = 0.75x - 0.25x^3, x \in (-1,1)$
- (C) $F(x) = 0.5 - 0.25x^2, x \in (-1,1]$
- (D) $F(x) = 0.5x^2 - 0.2x^3, x \in (-1,1]$

32) Perimeter of the cardioid $r = a(1 - \cos \theta)$ is

- (A) $6a$
- (B) $7a$
- (C) $8a$
- (D) none of these

33) If $x_r = \cos\left(\frac{\pi}{3r}\right) + i \sin\left(\frac{\pi}{3r}\right), r \geq 0$, then $x_1 x_2 x_3 \dots \infty$ is

- (A) 1
- (B) i
- (C) $-i$
- (D) none of these

34) The set $[0,1]$ is

- (A) countable
- (B) not countable
- (C) finite
- (D) none of these

35) If a circle and the rectangular hyperbola $xy = c^2$ meet in four points t_1, t_2, t_3 and t_4 , then $t_1 \cdot t_2 \cdot t_3 \cdot t_4$ is equal to

- (A) -1
- (B) 1
- (C) c
- (D) c^2

36) The direction cosines of a line equally inclined to the coordinate axes, are

- (A) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
- (B) $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
- (C) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$
- (D) All of the above

37) The tangents at the extremities of a focal chord of the parabola intersect at

- (A) 45°
- (B) 60°
- (C) 90°
- (D) 180°

38) For equally spaced tabular data for $y = f(x)$, the error in (Newton's forward) linear interpolation

- (A) does not exceed $\frac{1}{8}$ times the second difference
- (B) Equal to $\frac{1}{8}$ times the second difference
- (C) exceeds $\frac{1}{8}$ times the second difference
- (D) none of these

39) The real part of $\tan(x+iy)$ is

- (A) $\frac{\cosh 2x}{\cosh 2x + \cos 2y}$
- (B) $\frac{\cos 2x}{\cosh 2x + \cos 2y}$
- (C) $\frac{\sinh 2x}{\cosh 2x + \cos 2y}$
- (D) none of these

40) Let \mathbb{C} be the set of complex numbers and let $d: \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{R}$ be defined by setting

$$d(z_1, z_2) = |z_1 - z_2|, \forall z_1, z_2 \in \mathbb{C},$$

Then d is a metric on

- (A) \mathbb{R} (B) \mathbb{C} (C) $\mathbb{C} \times \mathbb{C}$ (D) none of these

41) Let (X, d) be a metric space and let $\rho(x, y) = \min\{1, d(x, y)\}$, for all x, y in X , then ρ is a

- (A) metric on X having no relation with d
(B) metric on X equivalent to d
(C) not a metric on X (D) none of these

42) Let $S = \left\{ (-1)^n \frac{n}{n+1} : n \in \mathbb{N} \right\}$, then the set S

- (A) has a limit point (B) has no limit point
(C) is not bounded (D) none of these

43) $\lim_{n \rightarrow \infty} \frac{1}{n} \left[1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \dots + n^{\frac{1}{n}} \right]$ is

- (A) 0 (B) 1 (C) 2 (D) none of these

44) The sequence $\langle S_n \rangle$ defined by $S_1 = 1, S_{n+1} = \frac{4+3S_n}{3+2S_n}, n \in \mathbb{N}$

- (A) converges to 2 (B) diverges to $+\infty$
(C) converges to $\sqrt{2}$ (D) none of these

45) The series $\sum \frac{(-1)^n}{\sqrt{n}}$ is

- (A) conditionally convergent (B) absolutely convergent
(C) not convergent (D) none of these

46) Differentiation of $\tan^{-1} \frac{2x}{1-x^2}$ with respect to $\sin^{-1} \frac{2x}{1+x^2}$ is

- (A) 1 (B) $\frac{2}{1+x^2}$ (C) $\frac{1}{1+x^2}$ (D) 2

47) If $f(x) = \tan x$, then $f^n(0) = {}^n C_2 f^{(n-2)}(0) + {}^n C_4 f^{(n-4)}(0) - \dots$ is

- (A) $\sin \frac{n\pi}{4}$ (B) $\sin \frac{n\pi}{3}$ (C) $\sin \frac{n\pi}{6}$ (D) $\sin \frac{n\pi}{2}$

48) The set $S = \left\{ -1, 1, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}, \frac{1}{3}, \dots \right\}$ is

- (A) closed (B) open
(C) neither open nor closed (D) none of these

49) Solution of $(D^2 - DD' - 6D'^2)z = xy$, where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$ is

$$(A) z = \varphi_1(y + 3x) + \varphi_2(y - 3x) + \frac{1}{6}x^3y + \frac{x^4}{24}$$

$$(B) z = \varphi(y + 3x) + \frac{1}{6}x^3y$$

$$(C) z = \varphi(y - 3x) + \frac{x^4}{24}$$

- (D) none of these

- 50) If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$, then
 (A) $m = -7, n = 1$ (B) $m = 1, n = -7$
 (C) $m = n = 0$ (D) $m = n = 1$

51) The residue of $\frac{\sin z}{z^8}$ at $z = 0$ is
 (A) 0 (B) $-\frac{1}{7!}$
 (C) $\frac{1}{7!}$ (D) none of these

52) The Fourier series of the 2π -periodic function $f(x) = x + x^2, -\pi < x \leq \pi$ at $x = \pi$ converges to
 (A) π (B) 2π
 (C) π^2 (D) $\pi + \pi^2$

53) The equation $e^x - 4x^2 = 0$ has a root between 4 and 5. Fixed point iteration with iteration function $\frac{1}{2}e^{x/2}$
 (A) diverges (B) converges
 (C) oscillates (D) converges monotonically

54) For the differential equation $t(t-2)^2 y'' + ty' + y = 0, t = 0$ is
 (A) An ordinary point (B) A branch point
 (C) an irregular point (D) A regular singular point

55) The general solution of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is of the form
 (A) $u = f(x+iy) + g(x-iy)$
 (B) $u = f(x+y) + g(x-y)$
 (C) $u = cf(x-iy)$
 (D) $u = g(x+iy)$

56) If $f: [a, b] \rightarrow \mathbb{R}$ is continuous and monotonic function, then
 (A) f is Riemann integrable on $[a, b]$ (B) f is not Riemann integrable on $[a, b]$
 (C) f is Riemann integrable on \mathbb{R} (D) none of these

57) Let f and g be bounded functions defined on $[a, b]$ and let P be any partition of $[a, b]$. Then,
 (A) $U(P, f+g) \leq U(P, f) + U(P, g)$ (B) $U(P, f+g) \geq U(P, f) + U(P, g)$
 (C) $U(P, f+g) = U(P, f) + U(P, g)$ (D) $L(P, f+g) \leq L(P, f) + L(P, g)$

58) The point-wise limit of sequence of real-valued function

$$f_n(x) = \sin x + \frac{x}{n}, \quad \forall x \in \mathbb{R}$$

 (A) $f(x) = 0, \quad \forall x \in \mathbb{R}$ (B) $f(x) = \begin{cases} 0, & x = 0 \\ 1, & \text{else} \end{cases}$
 (C) $f(x) = \sin x, \quad \forall x \in \mathbb{R}$ (D) does not exists

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$$f(x, y) = x^3y + e^{xy^2}.$$

Then, which one of the following is correct?

- (A) $f_{xy} > f_{yx}$ (B) $f_{xy} < f_{yx}$
 (C) $f_{xy} = f_{yx}$ (D) none of these

- In S_n the number of distinct cycles of length $r \leq n$ is
- (A) $\frac{1}{r} \frac{n!}{n-r!}$ (B) $\frac{n!}{n-r!}$
 (C) $\frac{(n-1)!}{n+r!}$ (D) None of these

- 72) If $X = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a ring under the addition and multiplication modulo 8, then this ring is
 (A) Commutative and integral domain (B) Integral domain
 (C) not an integral domain (D) None of these
- 73) In the ring M of 2×2 matrices over integers, consider the set $L = \left\{ \begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix}, a, b \in \mathbb{Z} \right\}$, then
 (A) L is not a left ideal (B) L is a left ideal
 (C) L is a right ideal (D) None of these
- 74) If V is an abelian group, then $\text{Hom}(V, V)$ is
 (A) a ring without unity (B) a ring with unity
 (C) not a ring (D) None of these
- 75) Which one is correct
 (A) Intersection of any family of subspaces of a vector space is a subspace
 (B) Intersection of any family of subspaces of a vector space is not a subspace
 (C) Union of any family of subspaces of a vector space is a subspace
 (D) None of these
- 76) The set of all solution of the differential equation

$$\frac{d^2y}{dt^2} + p \frac{dy}{dt} + qy = 0$$

 Where p and q are fixed functions of t ,
 (A) is a vector space over \mathbb{R} (B) is not a vector space over \mathbb{R}
 (C) may or may not be a vector space over \mathbb{R}
 (D) None of these
- 77) Vectors (ξ_1, ξ_2) and (η_1, η_2) in \mathbb{C}^2 are linearly dependent iff
 (A) $\xi_1\eta_2 = \xi_2\eta_1$ (B) $\xi_1^2\eta_2 = \xi_2^2\eta_1$
 (C) $\xi_1\xi_2 = \eta_1\eta_2$ (D) None of these
- 78) Let W be the set of real valued functions $y = f(x)$ satisfying $\frac{d^2y}{dx^2} + 4y = 0$, then dimension of W over \mathbb{R} is
 (A) 3 (B) 0 (C) 2 (D) None of these
- 79) Two vectors x, y in a Euclidean space are orthogonal iff
 (A) $\|x + y\|^2 = \|x\|^2 + \|y\|^2$ (B) $\|x + y\|^2 = \|x\|^2 + \|y\|^2 + 2\|x\|\|y\|$
 (C) $\|x + y\|^2 = \|x\|^2 - \|y\|^2$ (D) None of these
- 80) A linear transformation $T: \mathcal{V} \rightarrow \mathcal{V}$ is a projection if
 (A) $T^2 = T$ (B) $T^3 = T$ (C) $T^4 = T^3$ (D) None of these

1) All the eigen values of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ lie in the disc

- (A) $|\lambda + 1| \leq 1$ (B) $|\lambda - 1| \leq 1$
 (C) $|\lambda + 1| \leq 0$ (D) $|\lambda - 1| \leq 2$

92) $\iint_A xy dxdy$, where A is the domain bounded by x -axis, ordinate $x = 2a$ and the curve $x^2 = 4ay$ is

- (A) $\frac{a^4}{2}$ (B) $\frac{a^4}{3}$
 (C) $\frac{a^3}{3}$ (D) none of these

93) The cyclic group $Z_{30} = \{0, 1, 2, \dots, 29\}$ under addition modulo 30 has

- (A) 7 subgroups (B) 8 subgroups
 (C) 9 subgroups (D) 10 subgroups

94) Let

$$f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}, \quad (x, y) \neq (0, 0).$$

Then

- (A) $f(x, y)$ is discontinuous at $(0, 0)$ (B) $f(x, y)$ is continuous at $(0, 0)$
 (C) $f_x(0, 0) = 1$ (D) $f_y(0, 0) = 1$

95) The rank of the following $(n+1) \times (n+1)$ matrix where a is real number

$$\begin{bmatrix} 1 & a & a^2 & \vdots & a^n \\ 1 & a & a^2 & \vdots & a^n \\ \dots & \dots & \dots & \vdots & \dots \\ 1 & a & a^2 & \vdots & a^n \end{bmatrix}$$

is

- (A) 1 (B) 2
 (C) n (D) Depends on ' a '

96) Consider the basis $S = (v_1, v_2, v_3)$ for \mathbb{R}^3 where $v_1 = (1, 1, 1)$, $v_2 = (1, 1, 0)$, $v_3 = (1, 0, 0)$ and let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation such that $Tv_1 = (1, 0)$, $Tv_2 = (2, -1)$, $Tv_3 = (4, 3)$. Then, $T(2, -3, 5)$ is

- (A) $(-1, 5)$ (B) $(3, 4)$
 (C) $(0, 0)$ (D) $(9, 23)$

97) The solution of equation $z = pq$, is

- (A) $z = \sqrt{ax} + \frac{1}{\sqrt{a}}y - b$ (B) $z = \sqrt{ax} - \frac{1}{\sqrt{a}}y + b$
 (C) $z = \sqrt{ax} + \frac{1}{2\sqrt{a}}y + b$ (D) $2\sqrt{z} = \sqrt{ax} + \frac{1}{\sqrt{a}}y + b$

98) The moment of inertia of solid sphere of radius r and mass m about its diameter is

- (A) $\frac{2}{5}mr^3$ (B) $\frac{2}{5}mr^2$
 (C) $\frac{2}{3}mr^2$ (D) none of these

99) Consider the function

$$f(x) = \begin{cases} x - \sin x, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

- (A) $f(x)$ is everywhere discontinuous (B) $f(x)$ is continuous at one point
(C) $f(x)$ is continuous more than one point but at countable points
(D) none of these

100) The function $f(x) = |x + 2|$ is not differentiable at a point

- (A) $x = 2$ (B) $x = -2$
(C) $x = -1$ (D) $x = 1$