SYLLABUS

M.A./M./Sc. I SEMESTER

ORDINARY DIFFERENTIAL EQUATIONS : MMM-1006

UNIT-1 Introduction, initial value problem, boundary value problem, linear dependence equations with constant as well as variable coefficient, Wronskian, variation of parameter, method of undetermined coefficients, reduction of the order of equation, method of Laplace's transform.

UNIT-2 Lipchilz's condition and Gron Wall's inequality, Picards theorems, dependence of solution on initial conditions and on function, Continuation of solutions, Non-local existence of solutions Systems as vector equations, existence and uniqueness of solution to systems and existence and uniqueness of solution for linear systems.

UNIT-3 Introduction, Strum-Liouvilles system, Green's function and its applications to boundary value problems, some oscillation theorems such as Strum theorem, Strum comparison theorem and related results.

UNIT-4 Introduction, System of first order equation, fundamental matrix, Non-homogeneous linear system, Linear system's with constant as well as periodic coefficients.

Books Recommended:

1. E.A. Coddington: An introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1991.
2. S.C. Deo, Y. Lakshminathan and V. Raghavendra: Text Book of Ordinary Differential Equation (Second Edition) Tata McGraw Hill, New Delhi (Chapters IV, VII and VIII).

Reference Books:

1. P. Haitman: Ordinary Differential Equations, Wiley, New York, 1964.
2. E.A. Coddington and H. Davinson: Theory of Ordinary Differential Equations, McGraw Hill, NY, 1955.

SYLLABUS

M.A./M./Sc. I SEMESTER

Advance Theory of Groups and Homological Algebra: MMM-1007

Unit-I:

Relation of conjugacy, conjugate classes of a group, number of elements in a conjugate class of an element of a finite group, class equation in a finite group and related results, partition of a positive integer, conjugate classes in S_n, Sylow's theorems, external and internal direct products and related results.

Unit-II:

Structure theory of finite abelian groups, subgroup generated by a non-empty subset of a group, commutator subgroup of a group, subnormal series of a group, refinement of a subnormal series, length of a subnormal series, solvable groups and related results, n-th derived subgroup, upper central and lower central series of a group, nilpotent groups, relation between solvable and nilpotent groups, composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.

Unit-III:

Direct products and direct sums of modules, natural injections into the direct sum and natural projections from direct product with their related results, diagonal and summing homomorphisms, injective representation of a module as a direct sum, decomposition of a module into direct sum of submodules, free modules with related results, exact sequences and short exact sequences with their related results.

Unit-IV:

Splitting sequences with related results, The Four lemma, The Five lemma, semi-exact sequences, derived module, lower (upper) sequence or chain (cochain)complex of modules, n-dimensional chain of modules and boundary operators, modules of n-dimensional cycles and n-dimensional boundaries of a chain complex of modules, n-dimensional homology (cohomology) module of a lower (upper) sequence, tensor product of modules with related results.

Recommended and Reference Books:

- I.N. Herstein: Topics in Algebra.
- Surjeet Singh and Qazi Zameeruddin: Modern Algebra.
- P.B. Bhattacharya, S.K. Jain, S.R. Nagpaul: Basic Abstract Algebra
- S.T. Hu: Introduction to Homological Algebra.

SYLLABUS

M.A./M./Sc. I SEMESTER

TOPOLOGY-I : MMM-1009

UNIT-1 Definitions and examples of topological spaces, Topology induced by a metric, closed sets, Closure, Dense subsets, Neighbourhoods, Interior, Exterior and boundary accumulation points and derived sets, Bases and subbases, Topology generated by the subbasis, subspaces and relative topology, Alternative methods of defining a topology in terms of Kuratowski closure operator and neighbourhood systems, Continuous functions and homomorphism.

UNIT-2 First and second countable spaces, Lindelof spaces, Separable spaces, Second countability and separability, Separation axioms, T_0,T_1,T_2,$T_{3.5}$,T_4 spaces and their characterizations and basic properties, Brysohn's lemma, Tietze extension theorem.

UNIT-3 Compact spaces and their basic properties, Separation of a space, Connected spaces, Connected sets in the real line, Totally disconnected spaces, Intermediate value theorem, path connected, Components, Path components, Locally connected spaces, Locally path connected spaces, Totally disconnected spaces, Continuous functions and connected sets.

UNIT-4 Product topology (finite and infinite number of spaces), Tychonoff product topology in terms of standard sub-base and its characterizations, Projection maps, Separation axioms and product spaces, Connectedness and product spaces, Compactness and product spaces (Tychonoll's theorem), Countability and product spaces.

Books Recommended:

1. James R. Munkres: Topology, A first course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

SYLLABUS

M.A. /M. Sc. I Semester
Advanced Complex Analysis: MMM-1010

UNIT-I
Curves in the complex plane; Properties of complex line integrals; Fundamental theorem of line integrals (or contour integration); Simplest version of Cauchy's theorem; Cauchy-Goursat theorem; Symmetric, starlike, convex and simply connected domains; Cauchy's theorem for a disk; Cauchy's integral theorem; Index of a closed curve; Advanced versions of Cauchy integral formula and applications; Cauchy's estimate; Morera's theorem (Revisited); Riemann's removability theorem; Examples.

UNIT-II
Convergence of sequences and series of functions; Weierstrass' M-test; Power series as an analytic function; Root test; Ratio test; Uniqueness theorem for power series; Zeros of analytic functions; Identity theorem and related results; Maximum/Minimum modulus principles and theorems; Schwarz' lemma and its consequences; Advanced versions of Liouville's theorem; Fundamental theorem of algebra; Isolated and non-isolated singularities; Removable singularities; Poles; Characterization of singularities through Laurent's series; Examples.

UNIT-III
Calculus of residues; Residue at a finite point; Results for computing residues; Residue at the point at infinity; Cauchy's residue theorem; Residue formula; Meromorphic functions; Number of zeros and poles; Argument principle; Evaluation of integrals; Rouche's theorem; Mittag-Leffer expansion theorem; Examples.

UNIT-IV
Introduction and preliminaries; Conformal mappings; Special types of transformations; Basic properties of Möbius maps; Images of circles and lines under Mobius maps; Fixed points; Characterizations of Möbius maps in terms of their fixed points; Triples to triples under Möbius maps; Cross-ratio and its invariance property; Mappings of half-planes onto disks; Inverse function theorem and related results; Examples.

Books Recommended

1. John B. Conway; Functions of One Complex Variable, Second Edition, Springer International Student-Edition, Narosa Publishing House, 1980.
2. Lars V. Ahlfors; Complex Analysis, McGraw-Hill Book Company, Inc., New York, 1986.
3. S. Ponnusamy;Foundations of Complex Analysis, Second Edition, Narosa Publishing House, 2005.

SYLLABUS
M.A/M.Sc. I SEMESTER
FUNCTIONAL ANALYSIS: MMM-1011

UNIT-I Normed spaces; Banach spaces, their examples and properties; Incomplete normed spaces; Open and Closed spheres in normed spaces; Denseness, separability and closedness; Completion of normed linear spaces ; Finite dimensional normed spaces and subspaces; Equivalent norms; Compactness and finite dimension; Riesz`s lemma.

UNIT-II Quotient spaces; Bounded linear operators and bounded linear functional with their norms and properties; Algebraic and topological (continuous) duals; Examples and properties of dual spaces, Weak convergence and strong convergence (convergence in norm); Reflexive normed spaces and different kinds of topologies; Properties of reflexive normed spaces.

UNIT-III Hahn-Banach theorems and their consequences (Analytic and Geometric forms); Pointwise and uniform boundedness; Uniform boundedness principle and its applications; Open and closed maps; Open mapping and closed graph theorems, their consequences and applications; Banach contraction theorem with its applications.

UNIT-IV Inner product space and examples; Parallelogram law; Polarization identity and related results; Schwartz and triangle inequalities; Orthogonality of vectors, Orthogonal complements and related results; Projection theorem and related results; Orthogonal projection and properties.

Text Book:

1. E. Kreyszig: Introductory Functional Analysis with Applications, John Willey, 1978.

Reference Books:

1. M. Thumban Nair: Functional Analysis: A First Course, Prentice Hall of India, New Delhi, 2002.
2. P.K. Jain, O.P. Ahuja and Khalil Ahmad: Functional Analysis, New Age International (P) Limited, Publishers, 1995.

SYLLABUS
M.A./M./Sc. II SEMESTER
MEASURE THEORY : MMM-2002

UNIT-1 Lebesgue outer measure, Measurable and non-measurable sets measurable functions, Borel Lebesgue measureablity.

UNIT-2 Measure and outer measure, Extensions of a measure, Uniqueness of extension, Completion of measure, integration of non-negative functions, the general integral.

UNIT-3 Riemann and Lebesgue integrals, The four derivatives, Lebesgue differentiation, The differentiation and integration, Measure spaces, Convergence in measure.

UNIT-4 The L^p-spaces, Convex functions, Jensen's inequality, Holder and Minkowski inequalities, Completeness of L^p.

Books Recommended:

1. H.L. Royden: Real Analysis, Macmillan, 1993.
2. P.R. Halmos: Measure Theory, Van Nostrand, Princeton, 1950.

Reference Books:

1. Inder .K. Rana: An Introduction to Measure and Integration, Narosa, 1997.
2. P.K. Jain and V.P. Gupta: Lebesgue Measure and Integration, New Age International, 1986.

SYLLABUS

M.A./M./Sc. II SEMESTER

PARTIAL DIFFERENTIAL EQUATIONS : MMM-2005

UNIT-1 Classification of seconds order partial differential equation, Laplace's equation solution by the method of separation of variables, Fourier series solution, Applications to two dimensional heat flow, Mean value formulas, Properties of Harmonic functions, Green's functions.

UNIT-2 Heat equation-solution by the method of separation of variables, Fourier series solution, Applications to one dimensional heat flow, Mean value formula, Properties of solutions.

UNIT-3 Wave equation-solution by the method of separation of variables, Fourier series solution, Solution by spherical means and Riemann method of solution, Applications to vibration of strings..

UNIT-4 Numerical solution of partial differential equations, The wave equation, One dimensional heat flow and Laplace's equation.

Books Recommended:

1. Elements of Partial Differential Equations by I.N. Sneddon, McGraw Hill Book Company, 1957.
2. Partial Differential Equations by Phoolan Prasad and Renuka Ravindran, Wiley Eastern Limited, 1987.
3. Numerical Methods in Science and Engineering by M.K. Venkatraman, The national Publ. Company, 1990.
4. Calculus of Variations by I.M. Gelfand and S.V. Formin, Prentice Hall, Inc., 1963.
5. Partial Differential Equations by L.C. Ivans, M Graduate Studies in Mathematics, Volume 19, AMS, 1968

SYLLABUS
M.A./M.Sc. II Semester
Advanced Linear Algebra: MMM-2007

UNIT I: Recall of vector space, basis, dimension and related properties, Algebra of Linear transformations, Vector space of Linear transformations L(U,V), Dimension of space of linear transformations, Change of basis and transition matrices, Linear functional, Dual basis, Computing of a dual basis, Dual vector spaces, Annihilator, Second dual space, Dual transformations.

UNIT II: Inner-product spaces, Normed space, Cauchy-Schwartz inequality, Pythagorean Theorem, Projections, Orthogonal Projections, Orthogonal complements, Orthonormality, Matrix Representation of Inner-products, Gram-Schmidt Orthonormalization Process, Bessel's Inequality, Riesz Representation theorem and orthogonal Transformation, Inner product space isomorphism.

UNIT III: Operators on Inner-product spaces, Isometry on Inner-product spaces and related theorems, Adjoint operator, selfadjoint operator, normal operator and their properties, Matrix of adjoint operator **,** Algebra of Hom(V,V), Minimal Polynomial, Invertible Linear transformation, Characteristic Roots**,** Characteristic Polynomial and related results,

UNIT IV: Diagonalization of Matrices, Invariant Subspaces, Cayley-Hamilton Theorem, Canonical form, Jordan Form. Forms on vector spaces, Bilinear Functionals, Symmetric Bilinear Forms, Skew Symmetric Bilinear Forms, Rank of Bilinear Forms, Quadratic Forms, Classification of Real Quadratic forms.

BOOKS RECOMMENDED

1. Kenneth Hoffman and Ray Kunze : Linear Algebra (second Edition)
2. Sheldon Alexer: Linear Algebra Done Right, Springer (Third Edition)
3. I.N. Herstein: Topics in Algebra

SYLLABUS
M.A./M./Sc. II SEMESTER
ALGEBRAIC TOPOLOGY MMM-2008

UNIT-1 Urysohn Metrization Theorem, Partitions of unity, local finiteness, The Nagota Metrization Theorem, para-compactnes, The Smirnov Metrization Theorem.

UNIT-2 Nets and filters, topology and convergence of nets, Hausdorffness and nets, compactness and nets, filters and their convergence, canonical way connecting nets to filters and vice-versa, Ultra filters and compactness.

UNIT-3 Homotopy, relative homotopy, path homotopy, homotopy classes, construction of fundamental groups for topological spaces and its properties.

UNIT-4 Covering maps, local homomorphism, covering spaces, lifting lemma, The fundamental group of circle, Torus and punctured plane, The fundamental Theorem of Algebra.

Books Recommended:

(A) For Units I, III and IV:

1. Topology, A first course by J.M. Munkres 1987, (relevant portion)

(B) For Units II: (relevant portions of the following books)

2. Elementary Topology by M.C. Gemignani
3. Elementary General Topology by Jheral O. Moore
4. Topology by J. Dugundji
5. Topology by Sheldon W. Daves
6. Topology by H. Schubert

SYLLABUS

M.A./M./Sc. II SEMESTER

DIFFERENTIABLE MAINIFOLDS: MMM-2010

UNIT-1 Charts, Atlases, Manifolds, Differentiable structure on a manifold, Smooth maps, Tangent vectors and Tangent space.

UNIT-2 Vector fields, Lie product of Jacobian of a smooth map, Integral curves on a manifold, One parameter group of a transformation.

UNIT-3 Cotangent spaces, pullback of l-form, Tensor fields, Differential forms, Exterior product and derivative, Exterior algebra.

UNIT-4 Connexion, parallelism, Geodesic, Covariant differentiation Torsion, Curvature, Structure equation of Cartan, Bianchi identities.

Books Recommended:

1. Differentiable Manifolds: K. Matsushima.
2. Lecture Notes on Differentiable Manifolds: S.I. Husain

SYLLABUS
M.A./M./Sc. III SEMESTER
MECHANICS : MMM-3003

UNIT-1 General force system, euipollent force system, equilibrium conditions, Reduction of force systems, couples, moments and wrenches, Necessary and sufficient conditions of rigid bodies, General motion of rigid body, Moments and products of inertia and their properties, Momental ellipse, Kinetic energy and angular motion of rigid bodies.

UNIT-2 Moving frames of references and frames in general motion, Euler's dynamical equations, Motion of a rigid body with a fixed point under no force, Method of pointset Constraints, Generalized coordinates, D'Alembert's principle and Lagrange's equations, Applications of Lagrangian formulation.

UNIT-3 Hamilton's principle, Techniques of calculus of variations, Lagrange's equations through Hamilton's principle, Cyclic coordinates and conservation theorems, Canonical equations of Hamilton, Hamilton's equations from variational principle, Principle of least action.

UNIT-4 Galilean transformation, Postulates of special relativity, Lorentz transformation and its consequences, Length contraction, Time dilation, Addition of velocities, variation of mass with velocity, Equivalence of mass and energy, Four dimensional formalism, Relativistic classification of particles, Maxwell's equations and their Lorentz invariance.

Books Recommended:

1. J.L. Synge and B.A. Griffith: Principle of Mechanics, McGraw-Hill Book Company (1970) (relevant portion only).
2. H. Goldstein: Classical Mechanics: Second Edition, Narosa Publishing House (1980), (relevant portion only).
3. Zafar Ahsan: Lecture Notes on Mechanics, Department of Mathematics, AMU, (1999), (Chapters III-VI).

SYLLABUS

M.A./M./Sc. III SEMESTER

NONLINEAR FUNCTIONAL ANALYSIS: MMM-3005

Total Lectures: 48

UNIT-I:

Fixed Point Theorems: Banach contraction theorem and its extensions, namely, Boyd and Wong theorem, Caristi's fixed point theorem.

Set-Valued Maps: Introduction to set-valued maps, definitions and examples; Lower and upper semi continuity and their characterizations and examples.

UNIT-II:

Set-Valued Maps (Continue): Hausdorff metric, H-continuity or Hausdorff continuity, Set-valued Lipschitz maps, Set-valued contraction maps; Nadler's fixed point theorem.

Ekeland's Variational Principle: Strong and weak forms of Ekeland's variational principle, converse of Ekeland's variational principle, Applications to Banach contraction theorem, Caristi's fixed point theorem; Takahashi's minimization theorem.

UNIT-III:

Geometry of Banach Spaces: Strict convexity and modulus convexity; Uniform convexity; Duality mapping; Smoothness, Best approximation in Banach spaces, Retraction mappings.

UNIT-IV:

Iterative Methods for Fixed Points: Demiclosed Principle, Picard iterative method; Mann iterative methods, Ishikawa iterative method; Helpern iterative method; Browder iterative method.

Book Recommended:

1. Q.H. Ansari, Metric Spaces: Including Fixed Point Theory and Set-valued Maps, Narosa Publishing House, New Delhi, 2010.
   Sections 7.1, 7.2, 7.3, 8.1, 8.2, 8.3, 9.1, 9.2, 9.3 for unit I and II.
2. Q.H. Ansari, Topics in Nonlinear Analysis and Optimization, World Education, Delhi, 2012.
   Chapter 2: Sections 2.4, 2.5, 2.6, 2.7 for unit III.
3. S. Almezel, Q.H. Ansari and M.A. Khamsi, Topics in Fixed Point Theory, Springer, New York, 2014.
   Chapter 8 for unit IV.

Reference Book:

1. V.I. Istratescu, Fixed Point Theory: An Introduction, D. Reidel Publishing Company, Dordrecht / Boston / London, 1981, ISBN 90-277-1224-7.
2. M.A. Khamsi and W.A. Kirk, Metric Fixed Point Theory, Academic Press, New York.
3. S.P. Singh, B. Watson and P. Srivastava, Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht / Boston / London, 1997, ISBN 0-7923-4758-7.
4. W.Takahashi, Nonlinear Functional Analysis, Fixed Point Theory and its Applications, Yokohama Publishers, Yokohama, Japan, 2000, ISBN 4-946552-04-9.

SYLLABUS
M.A./M./Sc. III SEMESTER

RIEMANNIAN GEOMETRY AND SUBMANIFOLDS : MMM-3007

UNIT-1 Partition of unity, paracompactness, Riemannian matrix of a paracompact manifold, First fundamental form on a Riemannian manifold, Riemannian connexion, Riemannian curvature, Ricci and scalar curvature.

UNIT-2 Immersion, Imbedding, Distribution, Submanifold, Submanifold of Riemannian manifold, Sypersurfaces, Gauss and Weingarten formulae, Equation of Gauss, Coddazi and Ricci.

UNIT-3 Complex and almost manifolds, Nejenhuis tensor and integrability of a structure, Almost Hermitian, Kaehler and nearly Kaehler manifolds, Almost contact and Sasakian manifolds.

UNIT-4 Submanifolds of almost Hermitian manifolds, Invariant and Anti- Invariant distributions of a Hermitian manifold, C.R.-submanifolds of Kaehler and nearly Kaehler, Generic and slant submanifolds of Kaehler manifold.

Books Recommended:

1. Riemannian Geometry: R.S. Mishra
2. Geometry of Submanifolds: B.Y. Chen
3. Foundation of Geometry (Volume I): S. Kobayashi and K. Nomizu
4. Lecture Notes on Differentiable Manifolds: S.I. Husain

Optional Paper
Syllabus
M.A./M.Sc. III Semester
Variational Analysis and Optimization: MMM-3015

Unit 1: Convex Set, Hyperplanes, Convex function and its characterizations; Generalized convex functions and their characterizations, Optimality criteria, Kuhn-Tucker optimality criteria.

Unit 2: Subgradients and subdifferentials; Monotone and generalized monotone maps, their generalizations and their relations with convexity.

Unit 3: Variational inequalities and related problems, Existence and uniqueness results, Solution methods.

Unit 4: Generalized variational inequalities and related topics; Basic existence and uniqueness results.

Books:

1. Q.H. Ansari, C.S. Lalitha and M. Mehta, Generalized Convexity, Nonsmooth Variational and Nonsmooth Optimization, CRC Press, Taylor and Francis Group, Boca Raton, London, New York, 2014.

Optional Paper
Syllabus
M.A./ M.Sc. III-Semester
Wavelet Analysis
Paper Code: MMM-3016

Total Lectures: 48

Unit-I Gabor and Wavelet Transform

Fourier and inverse Fourier transform, Parseval identity, Convolution, Dirac delta function, Gabor transform, Gaussian function, Centre and width of Gaussian function, Time-frequency window of Gabor transform, Advantage of Gabor transform over Fourier transform, Continuous wavelet transform, Time-frequency window of wavelets, Discrete wavelet transform, Haar wavelet and its Fourier transform, Wavelets by convolution, Mexican hat wavelet, Morlet wavelet.

Unit-II Multiresolution Analysis and Construction of Wavelets

Parseval theorem for wavelet transform, Inversion formula of wavelets, Multiresolution Analysis, Decomposition and reconstruction algorithm, Filter coefficients and their properties, Wavelets and Fourier transform, Orthonormality in frequency domain, Numerical evaluation of scaling function and wavelets.

Unit-III Construction of Wavelets and its Applications

Cardinal B-splines and spline wavelets, Franklin wavelets, Battle-Lemari_e wavelets, Daubechies wavelets, Application of wavelets in Image processing, Wavelet packets, Best basis, Image compression and denoising.

Unite-IV Concept of Frames and its Applications

Concept of Frames in Hilbert space, Properties and related theorems, Characterization of frames, Frame multiresolution analysis, Gabor frames, Wavelet frames, Wavelet frames by extension principles, Applications of tight frames in image deblurring.

Recommended Books:

1. C.K. Chui, An Introduction to Wavelets, Academic Press, New York, 1992.
2. I. Daubechies, Ten Lectures on Wavelets, CBS-NSF Regional Conferences in Applied Mathematics, SIAM, Philadelphia, 1992.
3. O. Christensen, An Introduction to Frames and Riesz bases, Birkh□auser, Boston, 2003.

Reference Books:

1. Y. Meyer, Wavelets: Algorithms and Applications, SIAM, Philadelphia, 1993.
2. L. Debnath, Wavelet Transforms and their Applications, Birkh□auser, Boston, 2002.
3. M.W. Frazier, An Introduction to Wavelets through Linear Algebra, Springer, New York, 1999.
4. M.K. Ahmad, Lecture Notes on Wavelet Analysis, Seminar Library, Department of Maths, AMU, 2015.

OPTIONAL
SYLLABUS
M.A./M./Sc. III SEMESTER
LATTICE THEORY AND ALGEBRAIC STRUCTURES : MMM-3017

Total Lectures: 48

UNIT-1 Lattices (12 Lectures)

Partially order sets, Lattices, Modular Lattice, Schreier's theorem, The chain conditions, Decomposition theorem for lattices with ascending chain condition, Independence, Complemented modular lattices, Boolean Algebras.

UNIT-2 Modules and Ideals (12 Lectures)

Generators, Unitary Modules, Chain conditions, Hilbert Basis Theorem, Noetherian Rings, Prime and Primary ideals, Representation of an ideal as intersection of primary ideals, Uniqueness Theorems, Integral dependence.

UNIT-3 Lie and Jordan Structures in Rings (12 Lectures)

Lie and Jordan ideals in ring R, Jordan simplicity of ring R, Lie structure of [R, R], Subring fixed by automorphism, Simple rings with involutions, Involution of second kind, Skew elements and related results.

UNIT-4 Homomorphisms and Derivations (12 Lectures)

Jordan Homomorphisms onto Prime rings, n–Jordan mappings, Derivations, Lie Derivations and Jordan derivations, Some results of Martindale, Herstein theorem on Jordan derivation.

Books Recommended:

1. Lectures in Abstract Algebra by Nathan Jacobson
2. General Lattice Theory by George Gratzer
3. Topics in Rings Theory by I.N. Herstein
4. Rings with Involutions by I.N. Herstein